LETTRE XIV.

=

Goldbach à Euler.

Sommaire. Mêmes sujets. Réponse à la lettre précédente.

Moscuae d. $\frac{18}{29}$ Nov. 1731.

Jam in litteris Cal. Jun. 1730 ad Cl. Bernoullium datis monueram formulas

$$A \cdots \frac{dx}{(x^a+1)^{\frac{1}{n}}},\quad B \cdots \frac{u^b du}{(u^c+1)^{\frac{1}{n}}},\quad C \cdots \frac{dv}{(v^e+v^f)^{\frac{1}{n}}}$$

pro iisdem haberi posse, et propterea me uti velle formula A, in qua duae tantum exponentes arbitrariae insunt, cum in binis reliquis formulis tres exponentes reperiantur.

In formula Tua, quam pro $\int dx \frac{(a-x)(b-x) \text{ etc.}}{(\alpha+x)(\beta+x) \text{ etc.}}$ adsignas, non video quid fiat denominatore $= 0$ in expressione $\frac{(a+a)(a+b) \text{ etc.}}{(\beta-a)(\gamma-a) \text{ etc.}}$, si $\beta = \alpha$, vel $\gamma = \alpha$ etc.

Quod attinet ad $\int (1-x^{\frac{1}{n}})^p dx$ non facile puto inventum iri integralem praeter casus, quos jam in praecedente epistola

mea expressi, si scilicet per casus integrabiles eos tantum intelligemus, qui vulgo dici solent, quodsi vero magis ad naturam rei quam ad usum, qui inter Mathematicos obtinuit, respiciamus, apparebit sane eodem jure, quo hujus differentialis $(1-x)^{\frac{1}{2}}dx$ integralis genuina statuitur $-\frac{2}{3}(1-x)^{\frac{3}{2}}$, posse etiam $(1-x^{\frac{1}{n}})^p dx$ quovis alio casu integrari; nam cum $\int (1-x^{\frac{1}{n}})^p dx$, ut notum est, resolvi possit in

$$A \ldots \left(x + \frac{p+n+1}{n+1} A x^{\frac{n+1}{n}} + \frac{p+n+2}{n+2} B x^{\frac{n+2}{n}} + \text{etc.}\right)(1-x^{\frac{1}{n}})^{p+1}$$

vel in

$$B \cdots \left(\frac{-n}{p+1} x^{\frac{n-1}{n}} + \frac{n-1}{p+n-1} A x^{\frac{n-2}{n}} + \frac{n-2}{p+n-2} B x^{\frac{n-3}{n}} + \text{etc.}\right)(1-x^{\frac{1}{n}})^{p+1}$$

vel in

$$C \cdots x - \frac{pn}{n+1} x^{\frac{n+1}{n}} + \frac{p \cdot p-1 \cdot n}{2 \cdot n+2} x^{\frac{n+2}{n}} - \frac{p \cdot p-1 \cdot p-2 \cdot n}{2 \cdot 3 \cdot n+3} x^{\frac{n+3}{n}} + \text{etc.}$$

ad inveniendam integralem nihil aliud requiritur, quam ut determinetur formula generalis summarum hujus seriei

$$\frac{-n}{p+1} x^{n-1} + \frac{n-1}{p+n-1} A x^{\frac{n-2}{n}} + \text{etc.}$$

per expressionem quae finita maneat, posito pro n numero quocunque non integro; qua ratione autem hujusmodi formula designari possit, in dissertatione mea dixi, sola differentia, quae inter integrales has novas et alias jam cognitas irrationales (v. gr. $(1-x)^{\frac{3}{2}}$) intercedit, haec est quod illae a Mathematicis nondum receptae, hae longo usu jam confirmatae sunt.

Caeterum aequatio $(1-x^{\frac{1}{n}})^p dx = dy$ facile reducitur ad hanc $dz = (p+1)\, z\, dv + n\,(1-z)\, v^{-1} dv$, in qua exponentes prioris n et p coëfficientium locum tenent. Vale mihique fave.

Goldbach.