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How Euler Did It

by Ed Sandifer

Propulsion of Ships

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Long before there were Fields Medals or Nobel Prizes, the great scientific academies of Europe regularly proposed problems, with lucrative cash prizes for the best solutions. There has been an ironic change in the order of events for prizes over the last three centuries. Then, the Academies would meet and decide which questions were important. They would announce the problems, and savants around Europe would make their best efforts. Winners would be announced in a big ceremony, and the losers' entries would be "burned before the assembled Academy."

The Paris Prize was the most coveted of these. In the early and mid 1700's, the Paris problems often involved ships and navigation. For 1727, they posed a problem on the masting of ships, how many masts to use and where in the ship to position the masts. A nineteen year old Euler wrote his essay in 1726, and the when the results were published in 1728, he had won first prize. This sparked a lifetime off-and-on interest in Euler in mathematical and physical problems involving ships and navigation. Euler wrote only about a dozen papers on the subject, but he wrote two major books, including his last book, *Scientia Navalis*, published in two volumes in 1773.

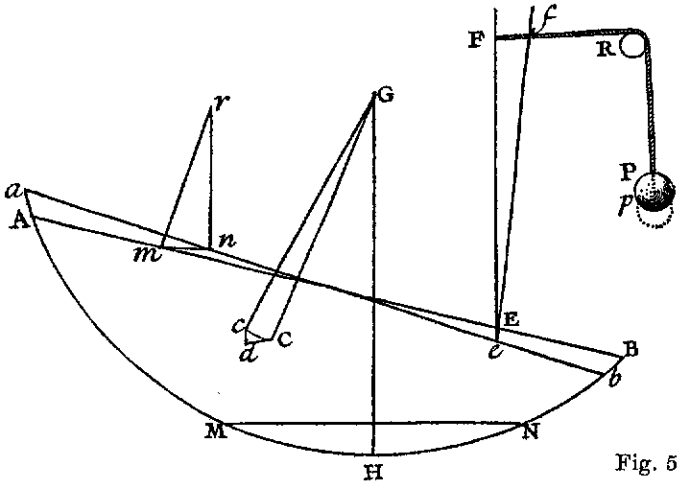
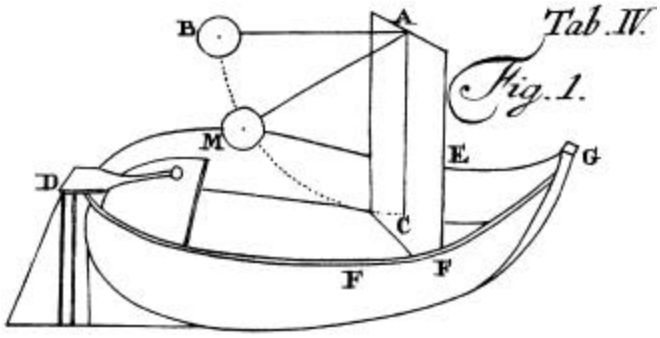


Fig. 5

An illustration from his paper for the 1727 Paris Prize is at the right.

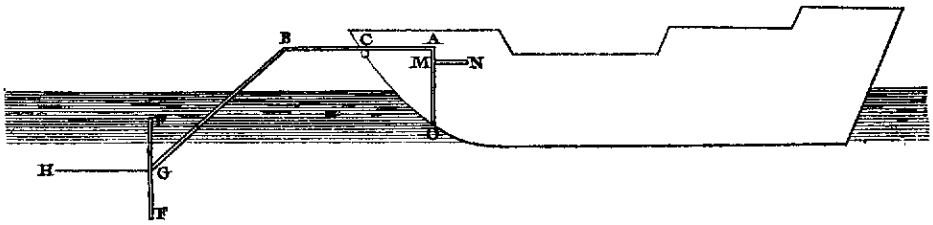
We leap ahead to Euler's fifth paper on nautical topics, E137, written in 1748 and published in 1750. Jakob Bernoulli's collected works appeared in 1744, and Euler noticed there an article where Bernoulli proposed a perpetual motion machine to propel a ship. The key idea is illustrated in the figure below.



Bernoulli proposed to erect a wall near the bow of the ship and to suspend a heavy weight from the top of the wall at the point marked A. He would lift the ball to the point B and let it drop. It would swing through M, strike the wall at C, and propel the boat forward. At the same time, he hoped, the ball would bounce back to B, from which it would fall again and propel the boat forward again. Obviously, Bernoulli never actually tested his design. It would have been a rough ride.

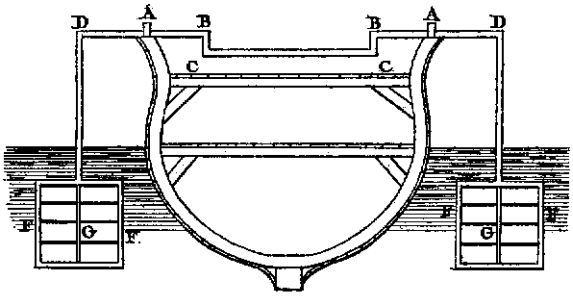
Euler didn't build the boat, either, but in this article he works through some calculations to show why it couldn't work.

Now let's move on to Euler's seventh nautical paper, E413, written in 1752 for the 1753 Paris Prize, but not published until 1771. It was titled "De promotione navium sine vi venti", or "On the movement of ships without the force of the wind." Euler's son Johan Albrecht published a French version of this same paper in 1766. In typical essay form, Euler proposes five successively more sophisticated ideas for propelling ships. His first idea, illustrated below, seems almost goofy:

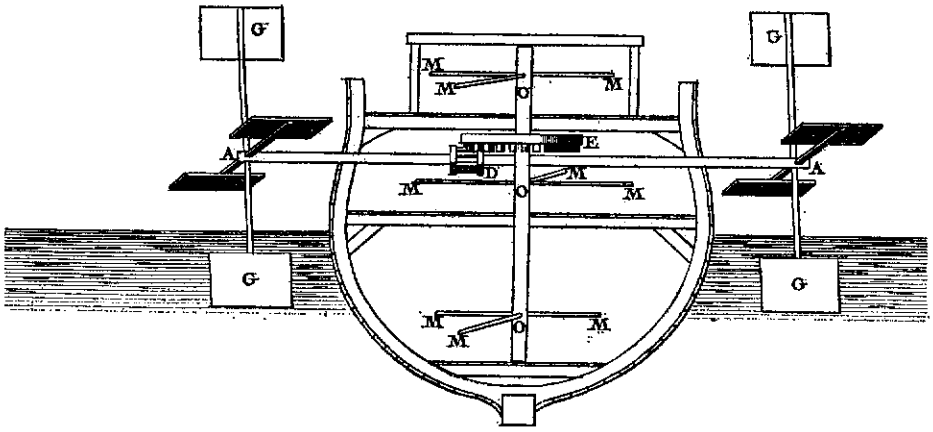


Here, we have a vertical surface, FF, attached to a bent support ACBG and suspended from the front of the boat. Workers inside the boat pull the system along the path MN, then lift it out of the water, push it forward and lower it back in to the water. It works kind of like a hoe. Euler decides it isn't really a very good idea.

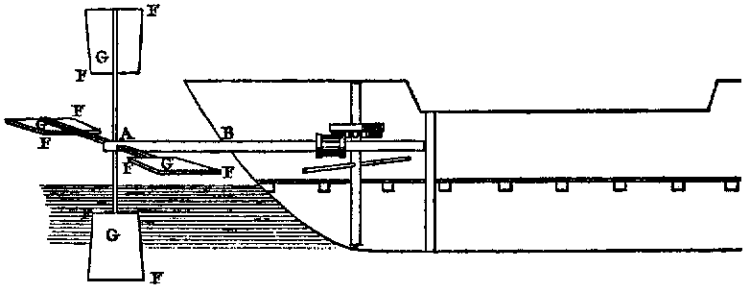
Euler's next idea, shown below, isn't very good, either. He proposes attaching paddles, FGF, to both sides of the ship and attaching the paddles to a shaft DABCCBAD. Workers on the deck can then turn the shaft and mechanically row the boat. It may seem like Euler is wasting his time on these silly ideas, but as he studies them, he is developing some equations of energy and fluid resistance that are quite useful.



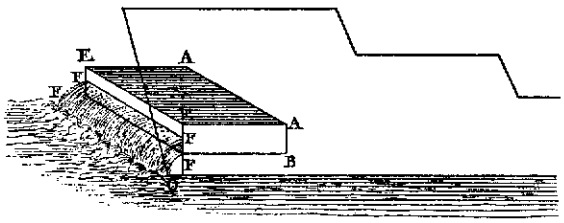
Euler's third idea is a modification of the one-paddle system we just described. He arranges four paddles around the shaft, instead of just one, and he gets a reasonable amount of energy by designing a system of cranks, all labeled M in the figure below. This is almost a 19th Century paddle wheel.



Euler's fourth idea also would not prove useful until the 19th Century. Shown below, he proposed extending a kind of fan in front of the ship, then turning it with the same kind of mechanism that he had used for the paddle wheel. This idea was patented in 1838 by the Swedish inventor John Ericsson. Ericsson had in mind that the propeller would be turned by a steam engine.



Euler's final idea is almost futuristic. His idea, pictured below, is to try to harness the motion of waves to move a ship. He proposes to put a trough, AEFB just above water level in the stern of the ship.



The idea is that the tops of the waves will be a little higher than the rim of the trough, and the trough will fill with water. Then, since the water in the trough is above the water level around the ship, the water can be drained out of the trough to propel the ship. In principle, the plan should work, but the effect would be very subtle, and there may be problems keeping the ship steady enough to make it work. However, in the 20th Century, electrical power generators were designed and built along the same principles, and they work just fine.

It seems that nobody tried to actually construct a ship that used any of Euler's ideas for propulsion. That is probably just as well, since, as he described them, they almost certainly would not have worked any better than Bernoulli's wall-striking idea would have worked. It is tempting to say, on the basis of these ideas, that Euler deserves credit for "inventing" the paddle wheel boat, the screw propeller and the wave-activated power systems. Not only would that be unfair to Fulton and Ericsson designed and built steam boats and screw propellers that actually worked, but to credit Euler would be an anachronistic view of history. Euler was using 18th Century ideas to try to solve 18th Century problems. His ideas didn't work. He had no reason to expect that, a hundred years later, steam engines would make ideas similar to his practical, and

even important. There is apparently no evidence that Ericsson or Fulton knew of Euler's ideas. They had to re-discover them to solve their 19th Century problems.

References

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How Euler Did It is updated each month.